

# TRR110 Workshop

$B \rightarrow 3h$  Amplitude Analysis in LHCb

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on behalf of the LHCb collaboration

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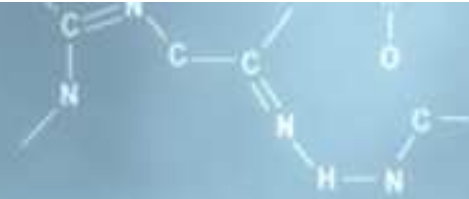
11 July 2018



**XUNTA  
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# Outline



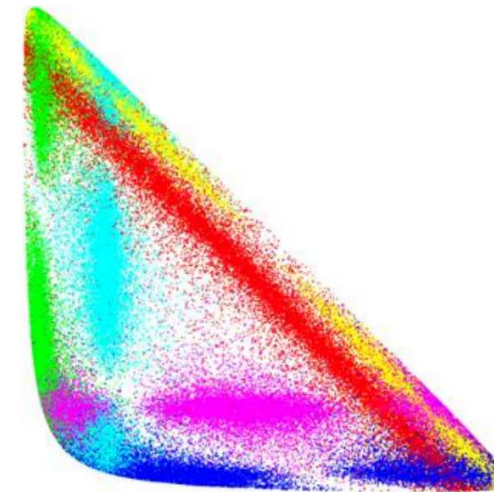
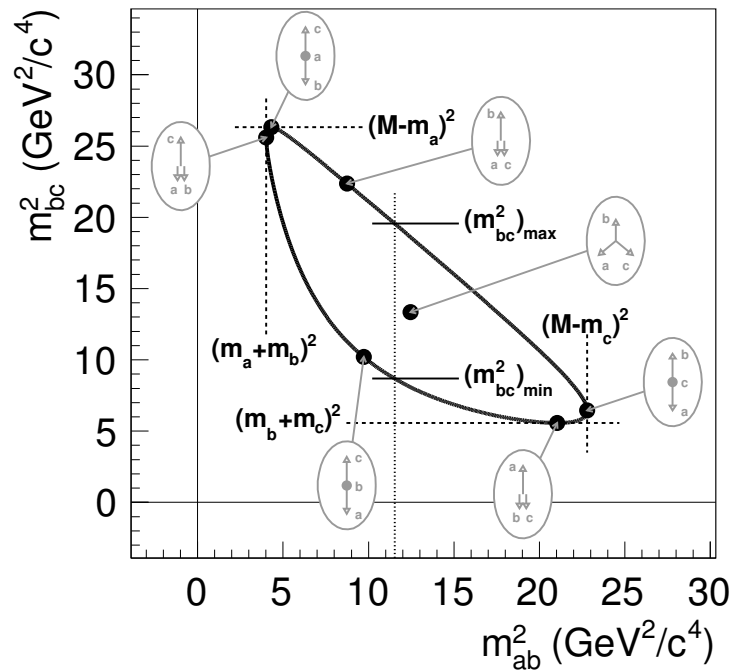
## 1. Manifestation of Direct $CP$ Violation in the Dalitz Plot

-Short/long-distance effects, rescattering

## 2. Recent Developments in Charmless Amplitude Analyses

-Rescattering, K-matrix, quasi-model-independent approaches to the  $S$ -wave

# Dalitz Plot



Toy MC Dalitz plot (DP)

Dalitz plot contains all kinematic and dynamic information of decay

Amplitude analysis one of the most powerful techniques

Extract amplitude-level information rather than amplitude-squared information

Interference between intermediate states allows measurement of relative magnitudes and phases

Resolve trigonometric ambiguities in phases that plague 2-body measurements

# Conditions for Direct $CP$ Violation

In charged  $B$  decays, presence of multiple amplitudes may lead to direct  $CP$  violation

$$A(B \rightarrow f) = \sum_i |A_i| e^{i(\delta_i + \phi_i)}$$

$$\bar{A}(\bar{B} \rightarrow \bar{f}) = \sum_i |A_i| e^{i(\delta_i - \phi_i)}$$

Strong phase ( $\delta$ ) invariant under  $CP$ , while weak phase ( $\phi$ ) changes sign under  $CP$

$$\mathcal{A}_{CP}(B \rightarrow f) \equiv \frac{|\bar{A}|^2 - |A|^2}{|\bar{A}|^2 + |A|^2} \propto \sum_{i,j} |A_i| |A_j| \sin(\delta_i - \delta_j) \sin(\phi_i - \phi_j)$$

3 conditions required for direct  $CP$  violation

At least 2 amplitudes

Non-zero strong phase difference,  $\delta_i - \delta_j \neq 0$

Non-zero weak phase difference,  $\phi_i - \phi_j \neq 0$

Source of weak phase differences comes from different CKM phases of each amplitude

# Short-Distance Contributions

Direct  $CP$  violation more complicated in  $B \rightarrow 3h$  decay channels compared to 2-body decays

There are at least 4 possible sources of strong phase

## 1. Short-distance contributions (quark level)

BSS mechanism, PRL **43** 242 (1979)

Tree contribution (a)

Penguin diagram (b) contains 3 quark generations in loop

$S$ -matrix unitarity,  $CPT$  require absorptive amplitude

If gluon in penguin is timelike (on-shell)

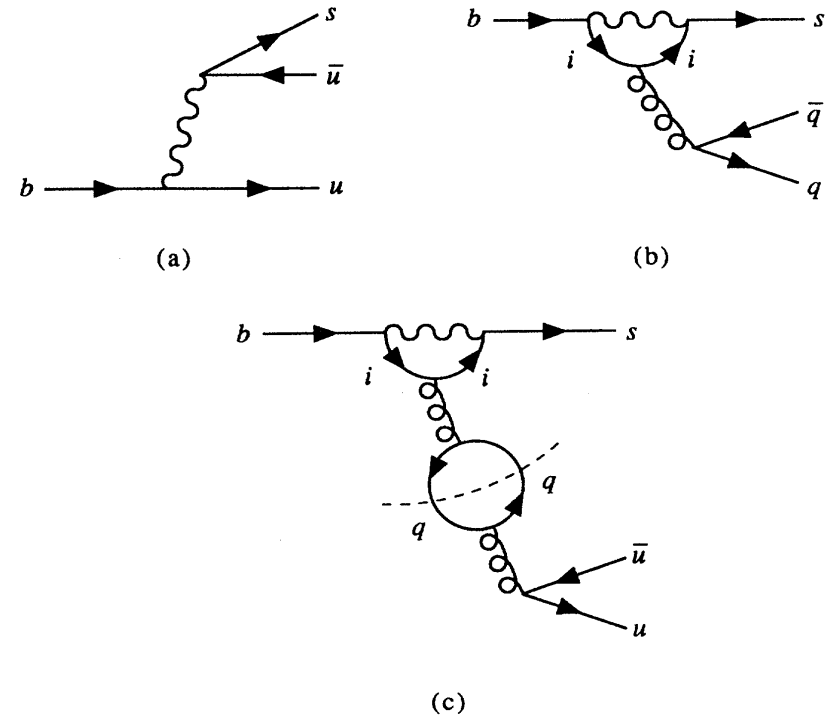
Momentum transfer  $q^2 > 4m_i^2$  where  $i = u, c$

Imaginary part depends on quark masses

Particle rescattering (c) generates a phase difference

$CP$  violation in 2-body processes caused by this effect

eg.  $B^0 \rightarrow K^+ \pi^-$



# Long-Distance Contributions

Remaining sources unique to multibody decays

Long-distance contributions ( $q\bar{q}$  level)

## 2. Breit-Wigner phase

Propagator represents intermediate resonance states

$$F_R^{\text{BW}}(s) = \frac{1}{m_R^2 - s - im_R\Gamma_R(s)}$$

Phase varies across the Dalitz plot

## 3. Relative $CP$ -even phase in the isobar model

$$A(B \rightarrow f) = \sum_i |A_i| e^{i(\delta_i + \phi_i)}$$

$$\bar{A}(\bar{B} \rightarrow \bar{f}) = \sum_i |\bar{A}_i| e^{i(\delta_i - \phi_i)}$$

Related to final state interactions between different resonances

# Manifestation of $CP$ Violation

Each source of strong phase leaves a unique signature in the Dalitz plot

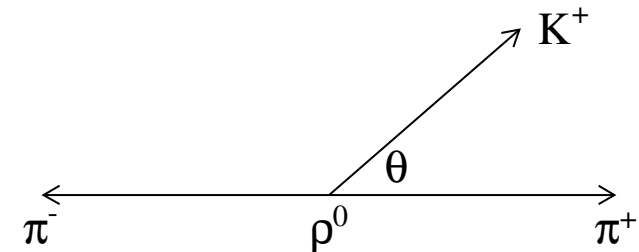
Illustrate with series of examples

Consider  $B^\pm \rightarrow K^\pm \pi^+ \pi^-$  with only 2 isobars

$B^\pm \rightarrow \rho^0 K^\pm$  and flat non-resonant (NR) component

$\rho^0$  lineshape a Breit-Wigner,  $F_\rho^{\text{BW}}$

$\rho^0$  is a vector resonance, so angular distribution follows  $\cos \theta$



$$A_+ = |a_+^\rho| e^{i\delta_+^\rho} F_\rho^{\text{BW}} \cos \theta + |a_+^{\text{NR}}| e^{i\delta_+^{\text{NR}}}$$

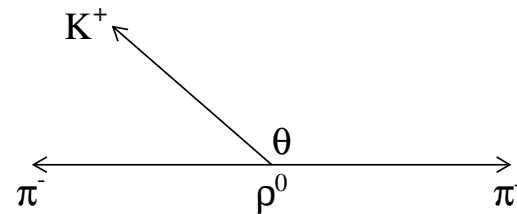
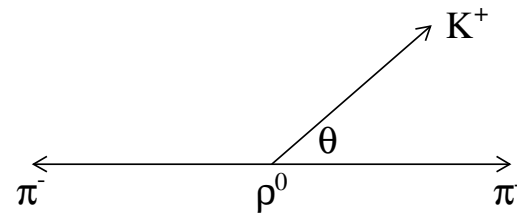
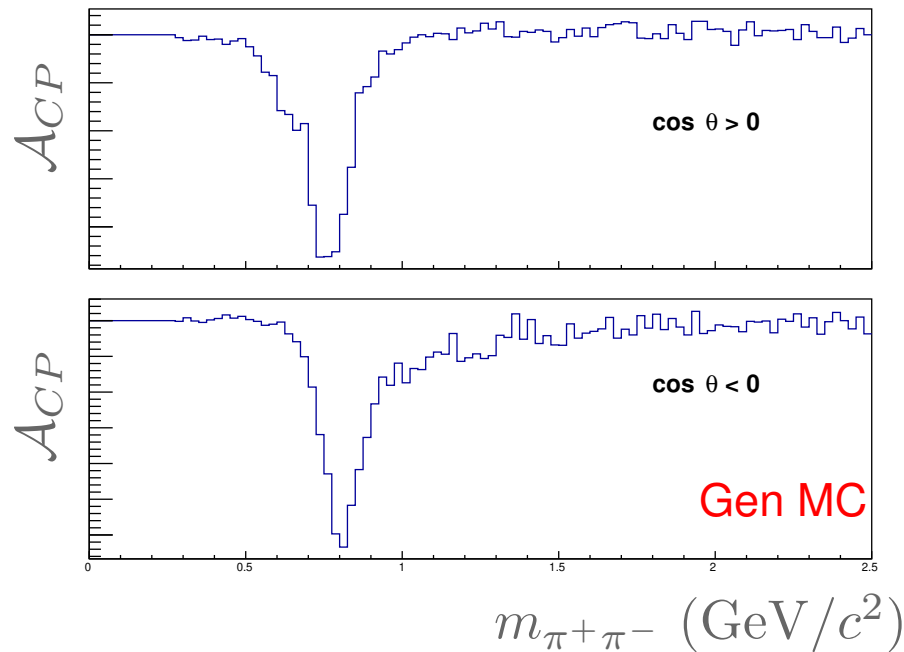
$$A_- = |a_-^\rho| e^{i\delta_-^\rho} F_\rho^{\text{BW}} \cos \theta + |a_-^{\text{NR}}| e^{i\delta_-^{\text{NR}}}$$

$$\begin{aligned} \mathcal{A}_{CP} &\propto |A_-|^2 - |A_+|^2 \\ &\propto (|a_-^\rho|^2 - |a_+^\rho|^2) |F_\rho^{\text{BW}}|^2 \cos^2 \theta \dots \\ &\quad - 2(m_\rho^2 - s) |F_\rho^{\text{BW}}|^2 \cos \theta \dots \\ &\quad + 2m_\rho \Gamma_\rho |F_\rho^{\text{BW}}|^2 \cos \theta \dots \end{aligned}$$

# Short-Distance Effects

$$\begin{aligned} \mathcal{A}_{CP} \propto & (|a_-^\rho|^2 - |a_+^\rho|^2) |F_\rho^{\text{BW}}|^2 \cos^2 \theta \dots \\ & - 2(m_\rho^2 - s) |F_\rho^{\text{BW}}|^2 \cos \theta \dots \\ & + 2m_\rho \Gamma_\rho |F_\rho^{\text{BW}}|^2 \cos \theta \dots \end{aligned}$$

Only depends on  $\rho$  resonance, maximum difference at  $\rho$  pole, quadratic in helicity



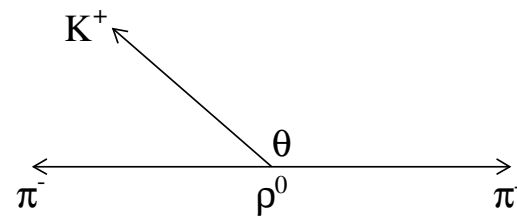
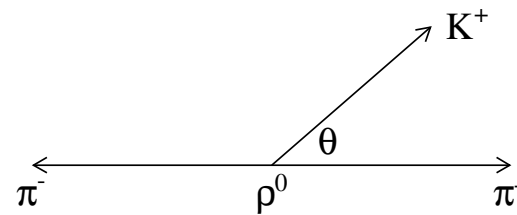
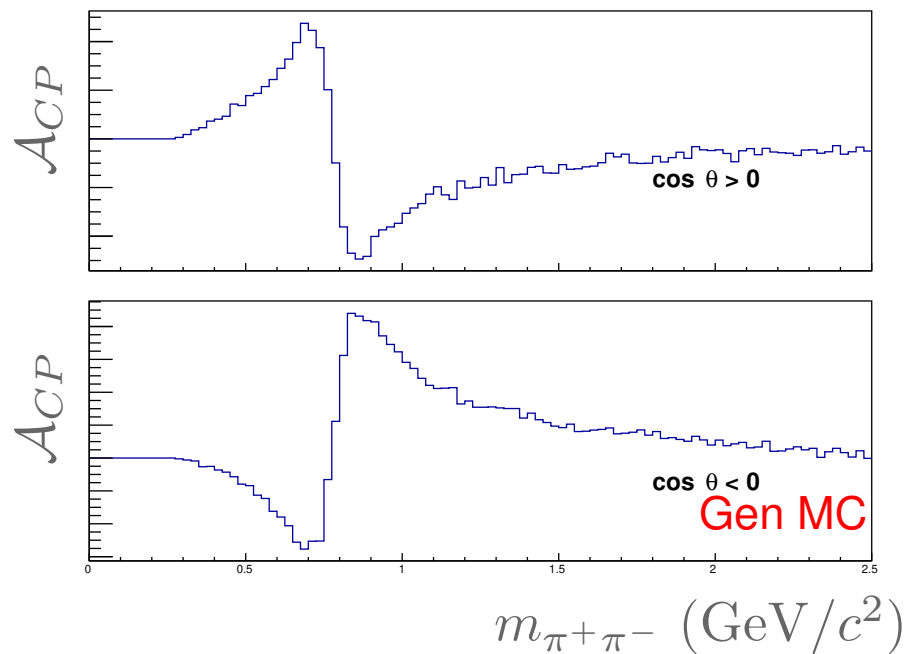
Only short-distance effects can create  $|a_+^\rho| \neq |a_-^\rho|$



# Long-Distance Effects

$$\begin{aligned} \mathcal{A}_{CP} &\propto (|a_-^\rho|^2 - |a_+^\rho|^2) |F_\rho^{\text{BW}}|^2 \cos^2 \theta \dots \\ &\quad - 2(m_\rho^2 - s) |F_\rho^{\text{BW}}|^2 \cos \theta \dots \\ &\quad + 2m_\rho \Gamma_\rho |F_\rho^{\text{BW}}|^2 \cos \theta \dots \end{aligned}$$

Interference term from real part of Breit-Wigner, zero at  $\rho$  pole, linear in helicity

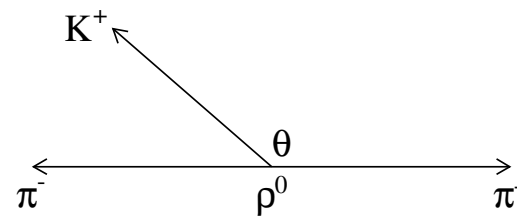
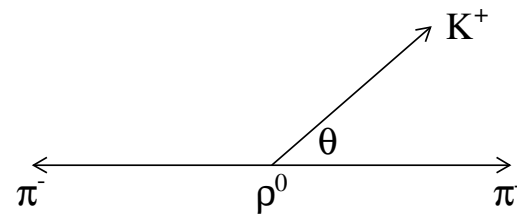
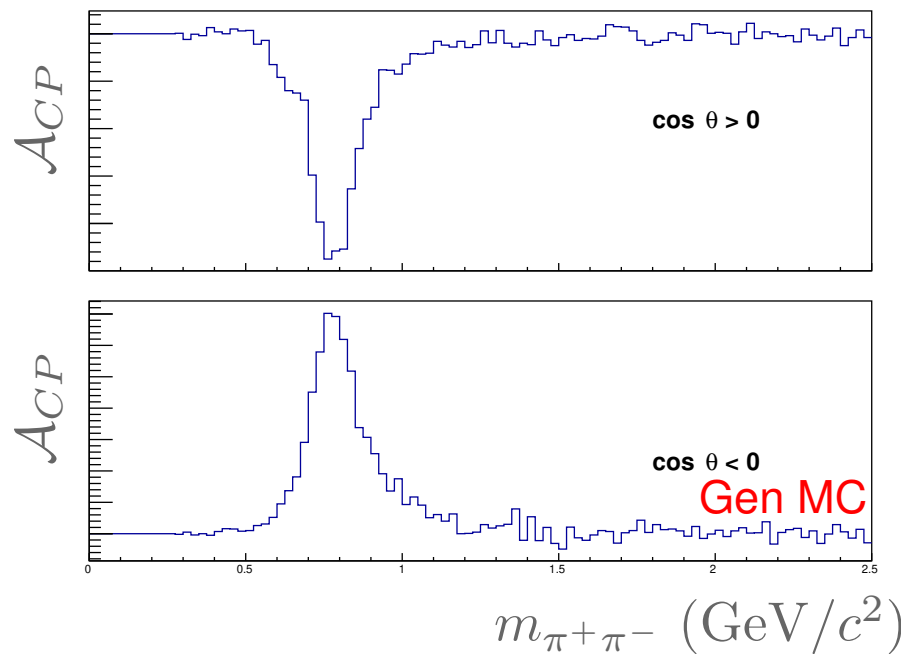


Caused by long-distance effects from final state interactions

# Long-Distance Effects

$$\begin{aligned} \mathcal{A}_{CP} \propto & (|a_-^\rho|^2 - |a_+^\rho|^2) |F_\rho^{\text{BW}}|^2 \cos^2 \theta \dots \\ & - 2(m_\rho^2 - s) |F_\rho^{\text{BW}}|^2 \cos \theta \dots \\ & + 2m_\rho \Gamma_\rho |F_\rho^{\text{BW}}|^2 \cos \theta \dots \end{aligned}$$

Interference term from imaginary part of Breit-Wigner, maximum at  $\rho$  pole, linear in helicity



Caused by long distance effects from Breit-Wigner phase and final state interactions

# Rescattering Contributions

Last source of strong phase

## 4. Final state $KK \leftrightarrow \pi\pi$ rescattering

Can occur between decay channels with the same flavour quantum numbers

$$\text{eg. } B^\pm \rightarrow K^\pm K^+ K^- \text{ and } B^\pm \rightarrow K^\pm \pi^+ \pi^-$$

$CPT$  conservation constrains hadron rescattering

For given quantum numbers, sum of partial widths equal for charge-conjugate decays

$KK \leftrightarrow \pi\pi$  rescattering generates a strong phase

Look into rescattering region

If rescattering phase in one decay channel generates direct  $CP$  violation in this region

Rescattering phase should generate opposite sign direct  $CP$  violation in partner decay channel

# LHCb Detector

$pp$  collisions

$b$  quark tends to forward/backward direction

Forward spectrometer

Vertex Locator (VeLo)

Precision tracking

$20 \mu\text{m}$  IP resolution

Tracking Stations (TT & T)

$\Delta p/p = 0.4\% - 0.6\%$

for  $5 - 100 \text{ GeV}$  tracks

Ring Imaging Cherenkov (RICH)

$K, \pi$  ID

Electromagnetic Calorimeter (ECAL)

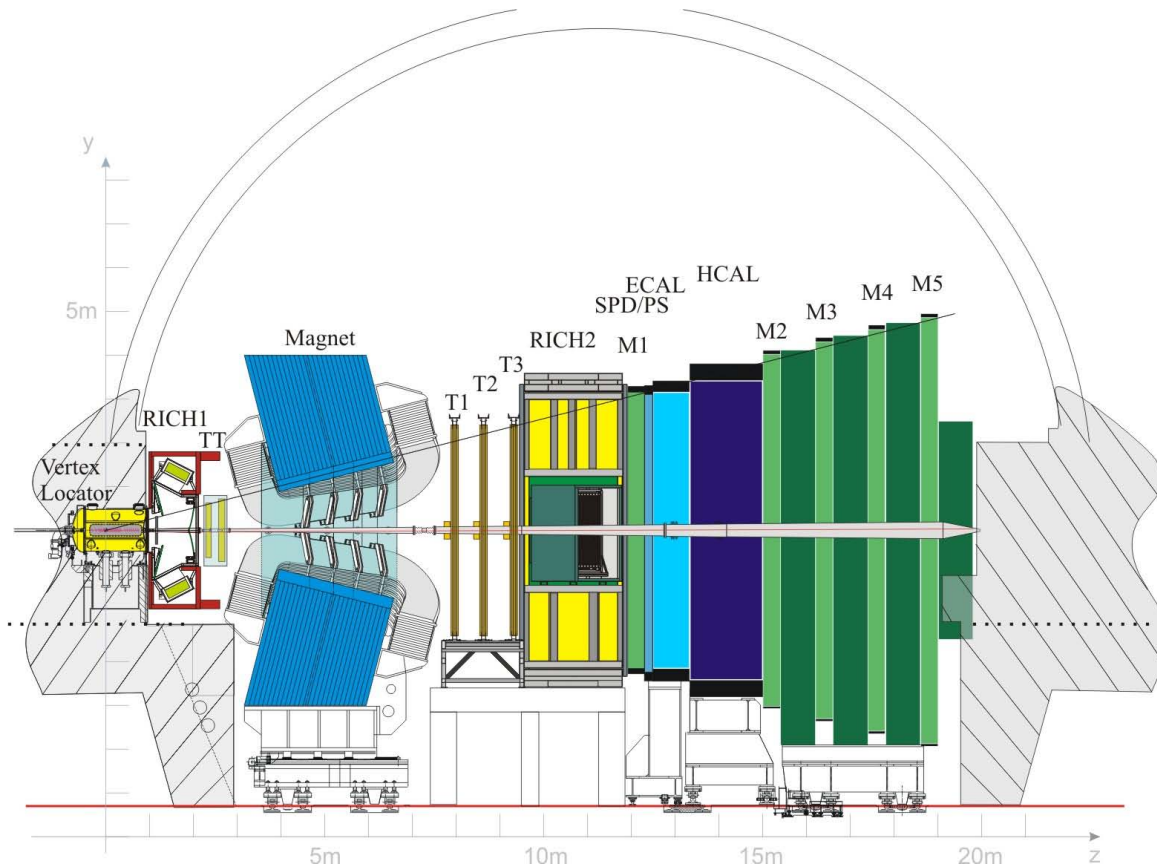
$e, \gamma$  ID

Hadronic Calorimeter (HCL)

Hadron ID

Muon Stations

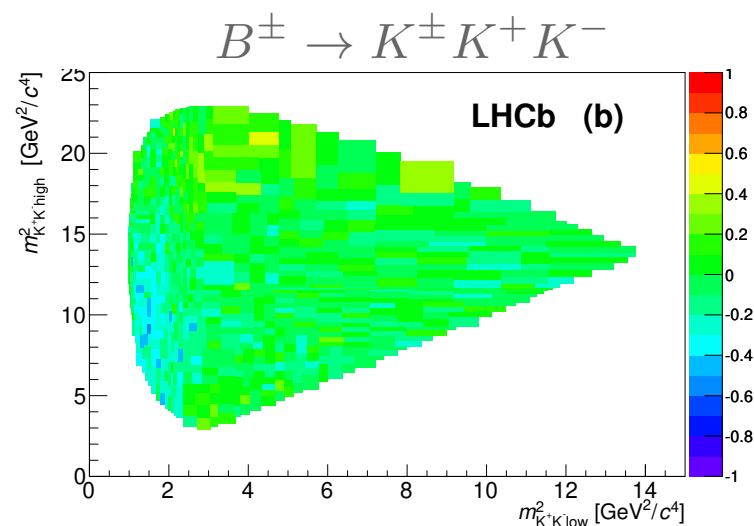
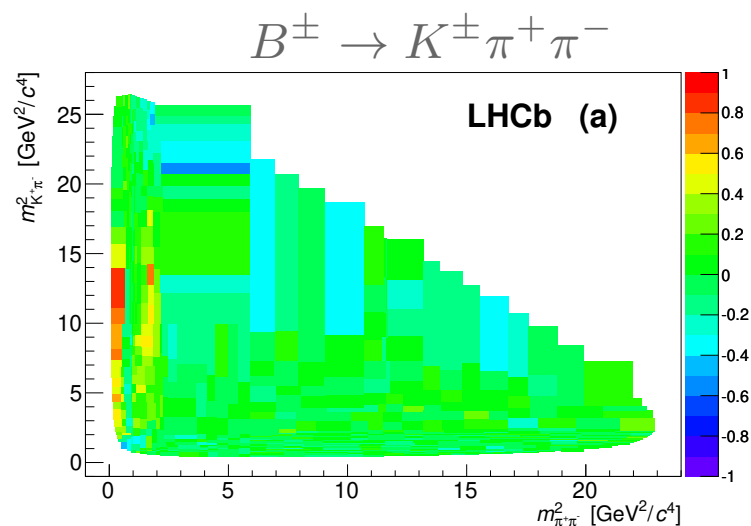
Dipole magnet polarity reversal



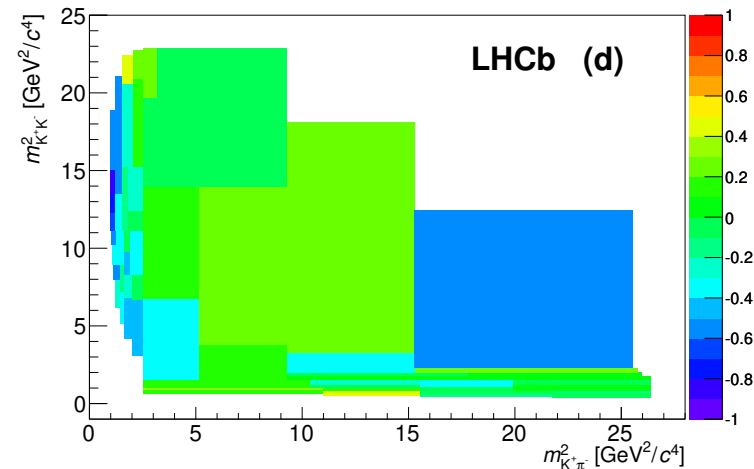
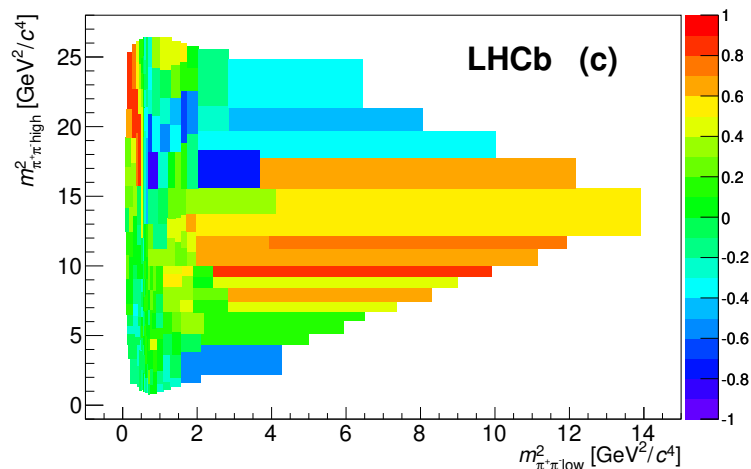
Data set:  $1 \text{ fb}^{-1}$  @  $7 \text{ TeV}$  and  $2 \text{ fb}^{-1}$  @  $8 \text{ TeV}$

$$B^\pm \rightarrow K^\pm h^+ h^-, \pi^\pm h^+ h^-$$

Observed large  $CP$  violating effects in the phase space, [Phys. Rev. D \*\*90\*\*, 112004 \(2014\)](#)



Penguin



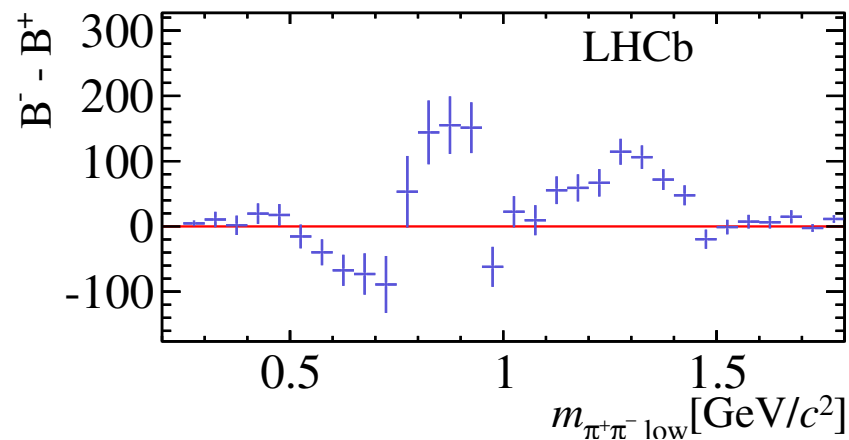
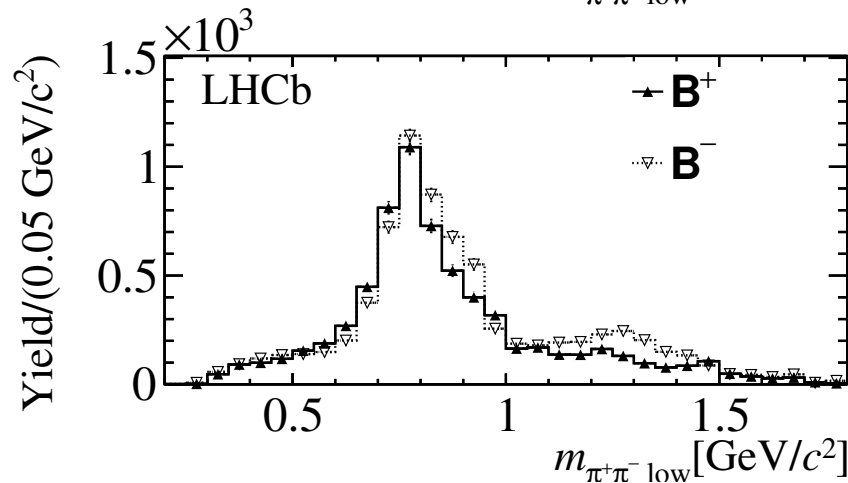
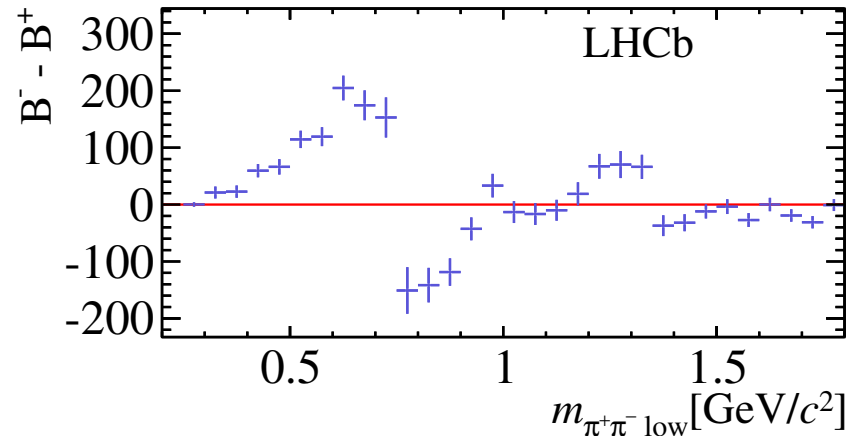
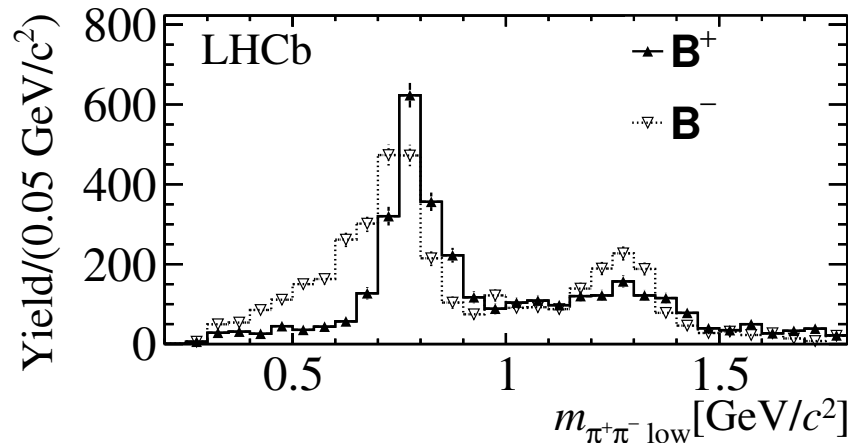
Tree

$$B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$$

$$B^\pm \rightarrow \pi^\pm K^+ K^-$$

# $CP$ Asymmetry by Interference

Project onto  $m_{\pi\pi}$  of  $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ , **Phys. Rev. D 90, 112004 (2014)**



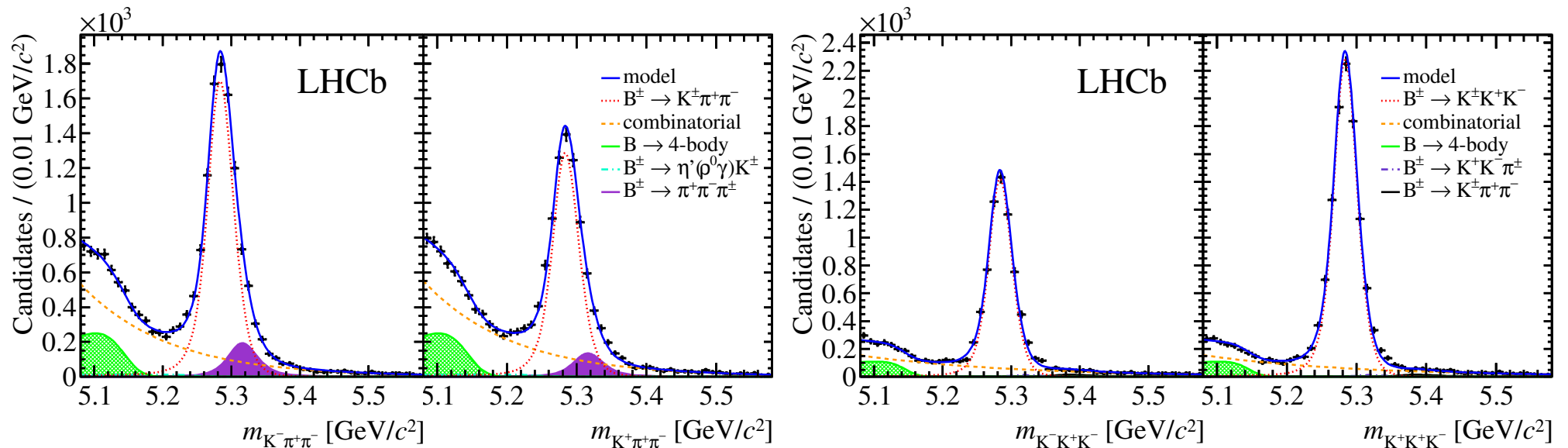
Sign-flip and zero around  $\rho^0$  pole,  $CP$  asymmetry may be dominated by real part of Breit-Wigner

# $CP$ Asymmetry by Rescattering

$\pi\pi \leftrightarrow KK$  rescattering region:  $1.0 - 1.5 \text{ GeV}/c^2$

$$B^- \rightarrow K^- \pi^+ \pi^- \quad B^+ \rightarrow K^+ \pi^+ \pi^-$$

$$B^- \rightarrow K^- K^+ K^- \quad B^+ \rightarrow K^+ K^+ K^-$$

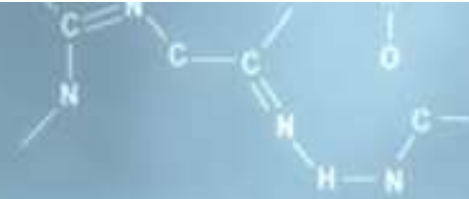


Clear opposite sign  $CP$  asymmetry in  $KK/\pi\pi$  - related channels

$KK \leftrightarrow \pi\pi$  rescattering would require this by  $CPT$  conservation

Phys. Rev. D **90**, 112004 (2014)

# Outline



## 1. Manifestation of Direct $CP$ Violation in the Dalitz Plot

-Short/long-distance effects, rescattering

## 2. Recent Developments in Charmless Amplitude Analyses

-Rescattering, K-matrix, quasi-model-independent approaches to the  $S$ -wave



# Rescattering Lineshape

Inspired by  $\pi\pi \leftrightarrow KK$  scattering in 2-body interactions

In the context of 3-body decays, production of one pair of mesons can affect the coupled channel

Attempt to account for this with phenomenological form factor

$$A(s) = \frac{\hat{T}}{1 + \frac{s}{\Delta_{PP}^2}}$$

Phys. Rev. D **92**, 054010 (2015)

Intended to describe the partonic interaction that produces  $\pi\pi$  and  $KK$  in 3-body final state

$\hat{T}$  is the observable amplitude related to the unitary  $S$ -matrix as,  $\hat{S} = 1 + 2i\hat{T}$

$$\hat{S}(s) = \begin{pmatrix} \eta(s)e^{2i\delta_{\pi\pi}(s)} & i\sqrt{1-\eta^2(s)}e^{i(\delta_{\pi\pi}(s)+\delta_{KK}(s))} \\ i\sqrt{1-\eta^2(s)}e^{i(\delta_{\pi\pi}(s)+\delta_{KK}(s))} & \eta(s)e^{2i\delta_{KK}(s)} \end{pmatrix}$$

# Rescattering Lineshape

Only off-diagonal elements are relevant for amplitude analysis

Use models for the phase shifts  $\delta_{\pi\pi}(s)$ ,  $\delta_{KK}(s)$  and inelasticity  $\eta(s)$

Phys. Rev. D **71**, 074016 (2005);

Phys. Rev. D **83**, 094011 (2011)

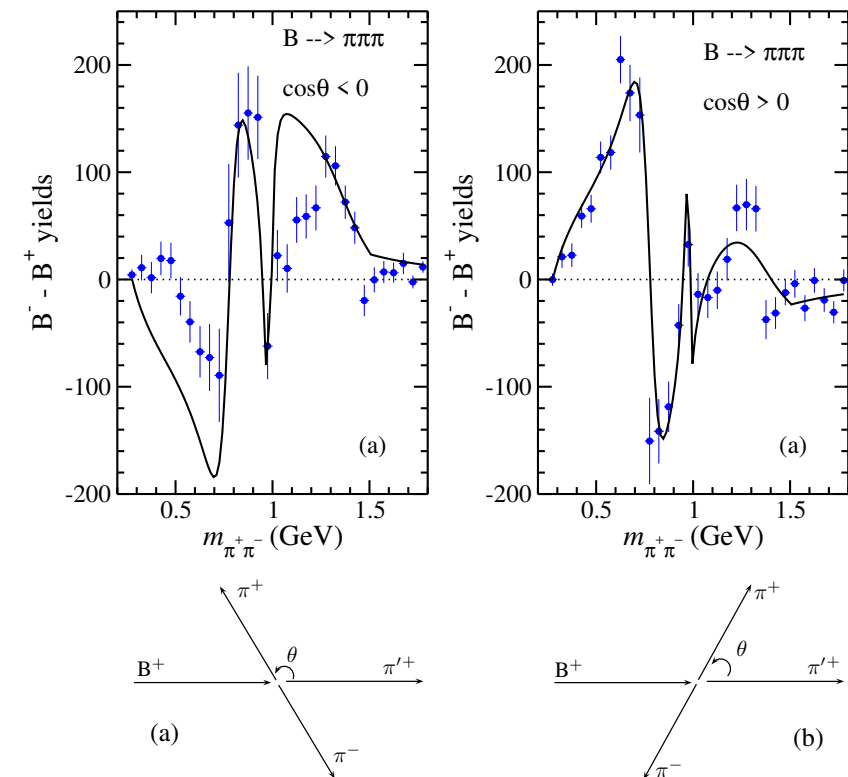
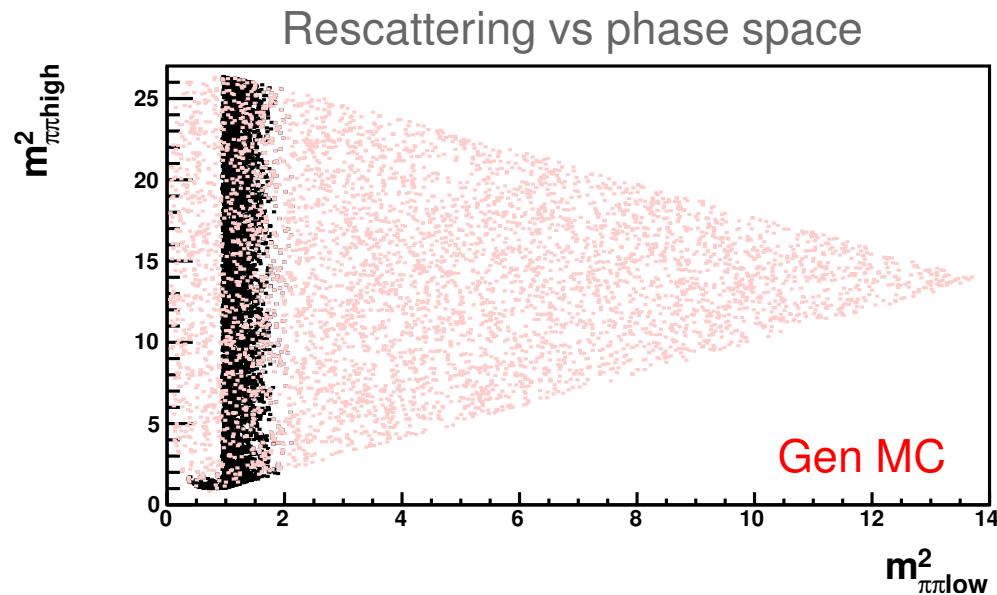
Also tested on LHCb asymmetry

$\rho$ ,  $f_0(980)$  considered in addition

Reproduces the main features

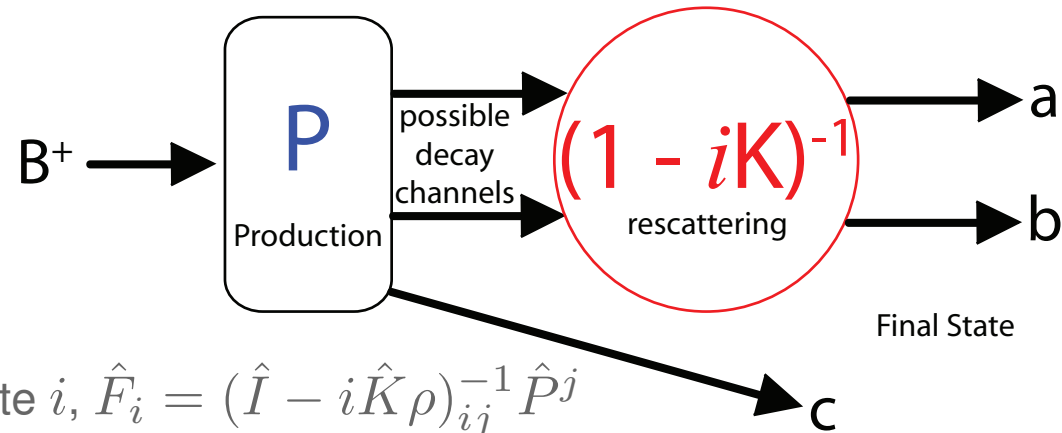
Exp: Phys. Rev. D **90**, 112004 (2014)

Th: Phys. Rev. D **92**, 054010 (2015)



# K-Matrix

From unitarity of the  $S$ -matrix, physical transition amplitude given by  $\hat{T} = (\hat{I} - i\hat{K}\rho)^{-1}\hat{K}$



For observed final state  $i$ ,  $\hat{F}_i = (\hat{I} - i\hat{K}\rho)_{ij}^{-1}\hat{P}_j$

$\hat{K}$  parametrised by summation of base mass poles and a slowly varying part for non-resonant

$$(\rho\hat{K})_{ij}(s) \equiv \sqrt{\rho_i\rho_j} \left( \sum_R \frac{g_i^R g_j^R}{m_R^2 - s} + f_{ij}^{\text{scat}} \frac{c - s_0^{\text{scat}}}{s - s_0^{\text{scat}}} \right) f_{A0}(s)$$

Parameters taken from scattering data

The production vector  $\hat{P}$  takes on an analogous form to  $\hat{K}$

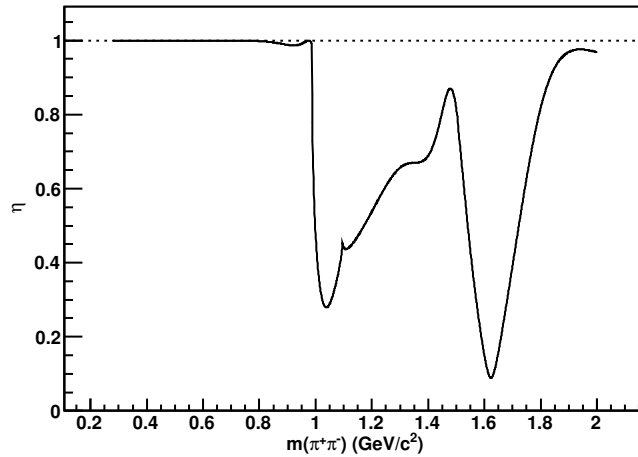
$$\hat{P}_j(s) \equiv \sum_R \frac{\beta_R^{\text{prod}} g_j^R}{m_R^2 - s} + f_j^{\text{prod}} \frac{c - s_0^{\text{prod}}}{s - s_0^{\text{prod}}}$$

$j: \pi\pi, KK, 4\pi, \eta\eta, \eta\eta'$ ;  $\beta_R^{\text{prod}}$  and  $f_j^{\text{prod}}$  are the complex free parameters of the model

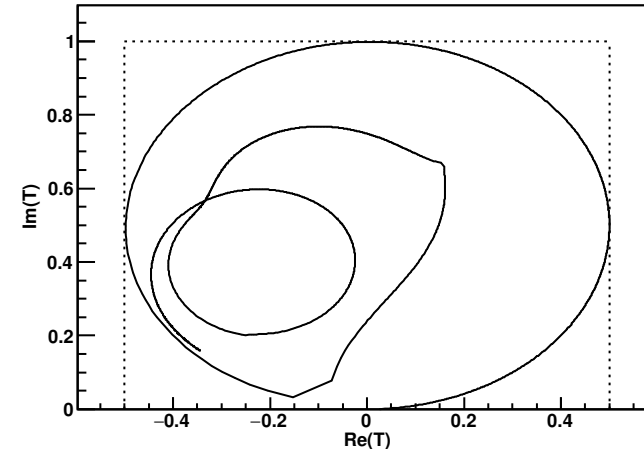
# K-Matrix

Elastic scattering on the physical boundary, inelastic scattering inside

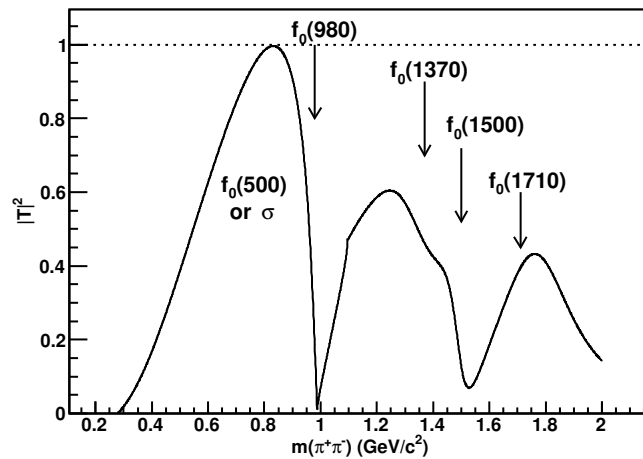
Inelasticity,  $\eta \equiv |2T - iI| = |S|$



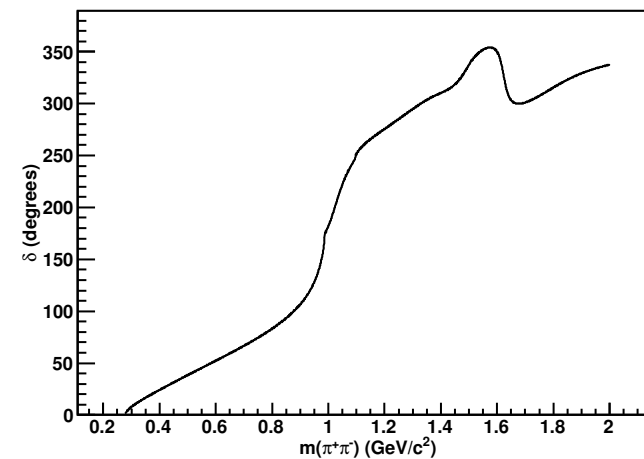
Transition amplitude T;  $S \equiv I + 2iT$



Transition amplitude intensity



Phase shift



Resonances don't necessarily manifest as Breit-Wigner structure

# Quasi-Model-Independent Approach

Construct spin-1 and spin-2 resonances with the isobar model as usual

Model  $\pi\pi$   $S$ -waves with adaptive binning method

Equal number of events in each bin

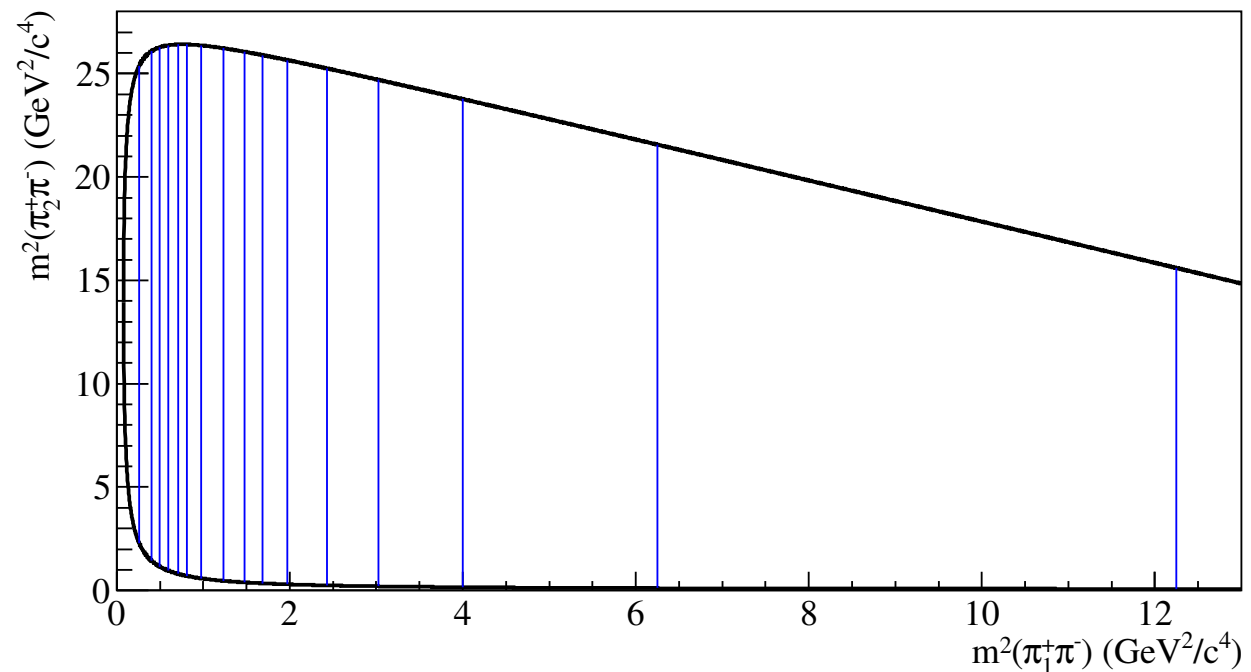
1D bins in  $m^2(\pi^+\pi^-)$ , 15 bins below charm veto, 2 bins above

In each bin,

float an amplitude and phase

81 free parameters in total

Bose-symmetric amplitude implied



# Quasi-Model-Independent Approach

Quasi-model-independent method

Reminiscent of partial wave analysis

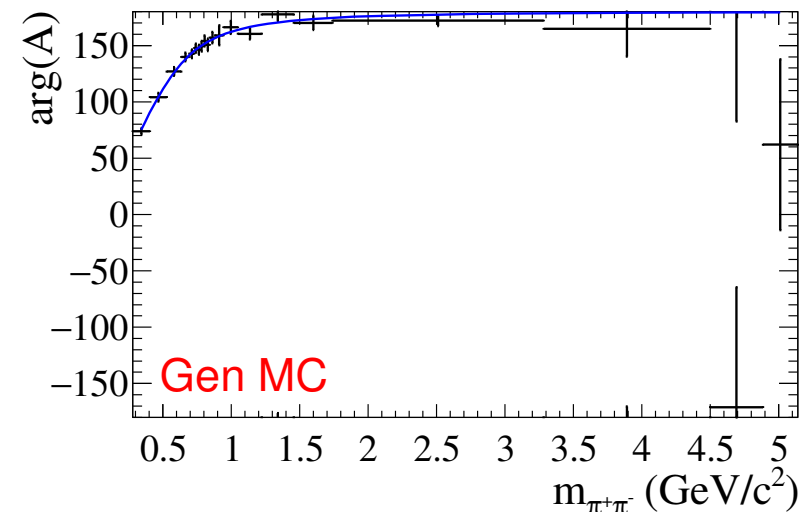
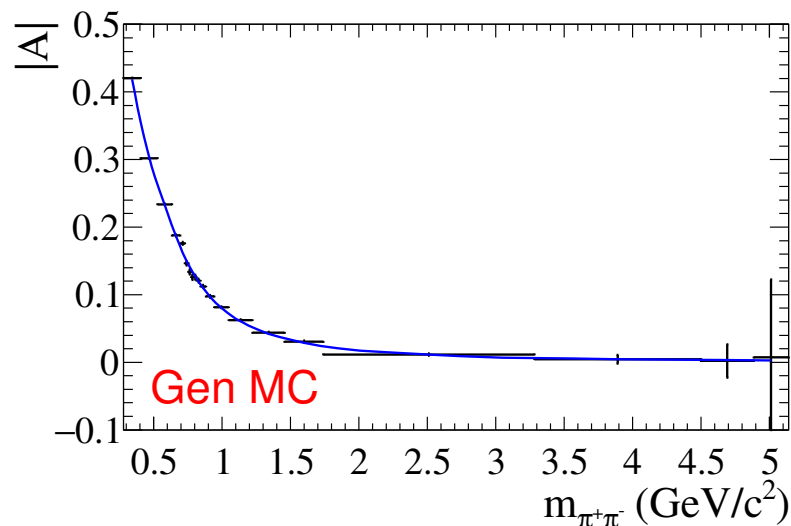
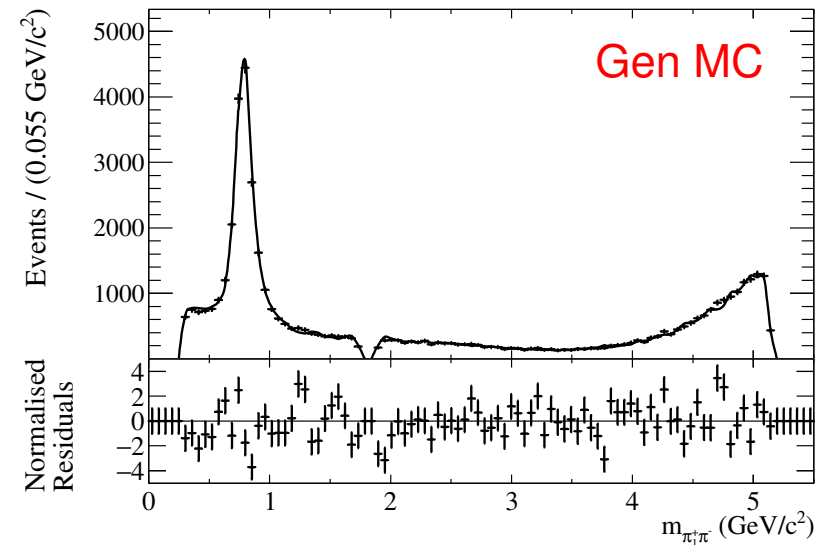
Divide the data into bins

Free magnitude and phase in each bin

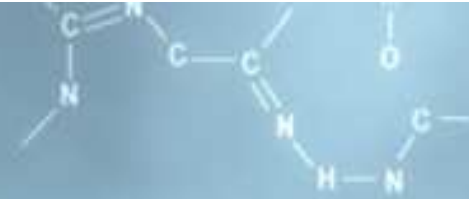
Data points: Fit results

Blue Curve: Generated  $f_0(500)$  Breit-Wigner

MC sample generated with  $\rho, f_0(500)$



# Summary



Large  $CP$  violating effects observed in the phase space

Arise from a variety of potential sources that need to be studied

Invoking  $CPT$  constraints to model rescattering effects between  $\pi\pi$  and  $KK$

Promising method to interface with the wealth of results from scattering experiments

Quasi-model-independent measurement of  $S$ -wave obtained directly from the data

Allows direct comparison with the two physics-motivated analytic  $S$ -wave analyses